The Analysis of Debt Capacity of BOT Projects

***Borliang Chen*[1]**

ABSTRACT

A theoretic model is established to determine the optimal capital structure and debt capacity of BOT projects with considering the bankruptcy cost and the possibility of default for the financial feasibility analysis of BOT projects. Dias et al (1995) proposed a model to calculate the debt value for promised repayment amount for a single period. This study extends the Dias' model to a multiple periods and for evaluation of project value. This modified model is named as the Dynamic Project Value Model (DPVM). The DPVM model is to evaluate the project value and to determine the repayment of the debt according to the amount of revenue and the variance of the expected revenue. In the other words, this model recommends that the more assurance on the revenue, the more possibility project company will pay the repayment as promised amount. An empirical case study of a university dormitory in the National United University (NUU) at Taiwan is considered to demonstrate the performance of the DPVM model. With case's parameters, the model is used to calculate the financial feasibility indices and the probability of bankruptcy of the projects. And, DPVM model can investigate the impact of the cost of bankruptcy and the probability of default on the optimal capital structure and debt capacity of the BOT projects. This DPVM dynamic model could provide an alternative for the evaluation of financial arrangement and financial risk analysis of BOT projects. This model also demonstrates that the debt providers are more conservative in financial analysis of debt ratio by comparing the financial analysis conducted by the project company.

Keywords: bankruptcy cost, probability of default, BOT, optimal capital structure, debt capacity

INTRODUCTION

Constant value of WACC (Weighted Average Cost of Capital), which to serve as project's discount rate, is widely used in the empirical financial feasibility analysis of BOT projects. The projects' discount rates play a very crucial role in determining the financial feasibility of the projects.

However, WACC is only an approximate estimate of the cost of the capital. Over optimism results will be often obtained in the calculation of financial feasibility indices with this fixed

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[1] *Associate Professor, Department of Civil Engineering, National United University, Miaoli, Taiwan 360.*

value of discount rate approach. That would sometimes lead to failure of BOT projects with highly risk in the implementation stage, such as the Airport MRT project at Taiwan.

In order to obtain more sophisticated financial arrangement on the debt repayment, a novel model of financial analysis with dynamic discount rates is proposed to analyze the financial feasibility of BOT projects. In this model, a dynamic discount rate, which is determined by considering volatility of annual revenue, could replace the WACC in the financial feasibility analysis of BOT projects.

Bakatjan et al (2003) and Zhang (2005) proposed a financial feasibility analysis model with WACC for BOT projects. Dias et al (1995) proposed a model to calculate the debt value for promised repayment amount in a single period. This study extends the Dias model to multiple periods and this extended model is named as the Dynamic Debt Repayment Model (DDRM). DDRM is a model with dynamic discount rate for each period in operation phase.

MODELING

DDRM dynamic model is established by considering the bankruptcy cost and the volatility of revenue in each period of operation stage. A theoretic analysis of deriving the full model is presented in the following sector. The final derivation of debt value in every single period, n, is shown in equation (12).

- The Dynamic Discount Rate Model in single period

The capital asset pricing model is in the form of the following equation:

$$E[\tilde{r}_i] = r_f + \beta_i \left(E[\tilde{r}_m] - r_f \right) \tag{1}$$

Where $E[\tilde{r}_i]$ is the expected return of company i. r_f is the risk-free return. β_i is the beta value of company i. $E[\tilde{r}_m]$ is the expected market return.

β_i can be determined by the following equation.

$$\beta_i = \frac{Cov(\tilde{r}_i, \tilde{r}_m)}{\sigma_m^2} \tag{2}$$

where $Cov(\tilde{r}_i, \tilde{r}_m)$ is the covariance of market return and firm i return. σ_m^2 is the variance of market return.

Combined the equation (1) and equation (2), we obtain

$$E[\tilde{r}_i] = r_f + \frac{Cov(\tilde{r}_i, \tilde{r}_m)}{\sigma_m^2}\left(E[\tilde{r}_m] - r_f\right) \tag{3}$$

Let

$$\tilde{R} = 1 + \tilde{r}_i ; \tilde{R}_m = 1 + \tilde{r}_m ; R_f = 1 + r_f \tag{4}$$

Substitute the equation (3) into equation (4), we obtain

$$\frac{E[\tilde{V}_1]}{V} = R_f + \left(E[\tilde{R}_m] - R_f\right)\frac{Cov(V_1, R_m)}{\sigma_m^2 V} \tag{5}$$

Where $E[\tilde{V}_1]$ is the expected firm return at the end of each period. V is the firm value at the beginning of each period. $Cov(V_1, R_m)$ is the covariance of firm value at the beginning of each period and the revenue at the end of each period.

Rearrange the equation (5), we find

$$V = \frac{E[\tilde{V}_1] - \lambda Cov(\tilde{V}_1, \tilde{R}_m)}{R_f} \tag{6}$$

$$\lambda = \frac{E[\tilde{R}_m] - R_f}{\sigma_m^2} \tag{7}$$

We obtain the debt value at the beginning of each period in the form of

$$D = \frac{E[\tilde{D}_1] - \lambda Cov(\tilde{D}_1, R_m)}{R_f} \tag{8}$$

Where D is the debt value at the beginning of each period. E[D1] is the expected value of debt value at the end of each period. Rm is the market return. Rf is risk free interest.

The expected value of debt value at the end of each period is in the form of [2]

$$E[\tilde{D}_1] = d_1[1 - F_X(d_1)] + (1 - b_v)\left[E[\tilde{X}][F_X(d_1) - F_X(b')] + \sigma_X{}^2[f_X(b') - f_X(d_1)]\right] \tag{9}$$

Where d1 is the amount of promised debt repayment at the end of each period. $F_X(d_1)$ is the accumulated probability of not full payment for the debt. b_v is the variable bankruptcy cost. $E[\tilde{X}]$ is the expected value of the revenue in each period. $F_X(b')$ is the accumulated probability for no payment to the loan providers. σ_X is the standard deviation of revenue in each period. $f_X(b')$ is the density function for no payment to the loan providers. $f_X(d_1)$ is the density function for not full payment for the debt.

The covariance is in the form of

$$Cov(\tilde{D}_1, \tilde{R}_m) = \left[(1-b_v)\left[F_X(d_1) - F_X(b')\right] + \left[b_f + b_v d_1\right] f_X(d_1)\right] Cov(\tilde{X}, \tilde{R}_m) \qquad (10)$$

where b_f is the fixed bankruptcy cost.

Substitute the equation (9) and (10) into equation (8), we have the debt value in the form of

$$D = \left(\frac{1}{R_f}\right)\left\{\begin{matrix} d_1\left[1-F_X(d_1)\right] + (1-b_v)\left[E[\tilde{X}]\left[F_X(d_1) - F_X(b')\right] + \sigma_X^{\,2}\left[f_X(b') - f_X(d_1)\right]\right] \\ -b_f\left[F_X(d_1) - F_X(b')\right] - \lambda Cov(\tilde{X}, \tilde{R}_m)\left[(1-b_v)\left[F_X(d_1) - F_X(b')\right] + \left[b_f + b_v d_1\right] f_X(d_1)\right] \end{matrix}\right\} \qquad (11)$$

- The Dynamic Discount Rate Model in multiple period

Extend the equation (11) , which is good for single period, into multiple period.

$$D_n = \frac{\left\{\begin{matrix} d_{1,n}\left[1-F_X(d_{1,n})\right] + (1-b_v)\left[E[\tilde{X}_n]\left[F_X(d_{1,n}) - F_X(b')\right] + \sigma_X^{\,2}\left[f_X(b') - f_X(d_{1,n})\right]\right] \\ -b_f\left[F_X(d_{1,n}) - F_X(b')\right] - \lambda Cov(\tilde{X}_n, \tilde{R}_m)\left[(1-b_v)\left[F_X(d_{1,n}) - F_X(b')\right] + \left[b_f + b_v d_{1,n}\right] f_X(d_{1,n})\right] \end{matrix}\right\}^n}{R_f^{\,n} \times d_{1,n}^{\,(n-1)}} \qquad (12)$$

where D_n is the debt value at the beginning of the nth period in the operation phase. $d_{1,n}$ is the debt value at the end of nth period in the operation phase. $E[\tilde{X}_n]$ is expected revenue in the nth period. $Cov(\tilde{X}_n, \tilde{R}_m)$ is the covariance of revenue and market return in the nth period.

The equation to calculate the dynamic debt interest rate in each period with considering the volatility of revenue, bankruptcy cost, and CAPM model is as shown in equation (2).

$$r_{D,n} = \frac{d_{1,n}}{D_{1,n}} - 1 \qquad (13)$$

Criteria

Three criteria were adopted in the study: (1) maximizing the project value that generates the optimal capital structure; (2) maximizing the debt value for obtaining the debt capacity; and (3) value at risk with 99% confidence level.

EMPIRICAL STUDY

The case of university dormitory of National United University (NUU) at Taiwan is to illustrate as an empirical study of this paper. It is a BOT project of dormitory.

Input parameters

Input parameters of the National United University dormitory BOT project are shown as Table 1.

Table1、Financial parameters of case study for PFEM and DDRM models

Item	Value	Remarks
Concession period	40	In year 2006~2046
Construction period	2	In year 2006~2008
Operation period	38	In year 2008~2046
Inflation rate	1.8%	
Annual growth rate of salary	2%	
Annual rent escalation	2%	
Income tax	25%	
Business tax	5%	
Grace period	2	
Repayment period	15	
Discount rate	6.50%	

Some parameters are required for dynamic model shown in Table 2. Table 3 shows the revenue in each year. These value is given by the engineering consultant firm.

Table2、Parameters for dynamic discount rate model

Risk free return (R_f)	1.0244
Market return (R_m)	**1.150**
Standard deviation of market return (σ_m)	**0.250**
Covariance between revenue and market return (ρ_x，R_m)	**0.700**

Table3、Revenues in operation period in NT$

1 year	**2 year**	**3 year**	**4 year**	**5 year**	**6 year**	**7 year**	**8 year**	**9 year**	**10 year**
31,906,005	32,594,242	33,297,714	34,015,048	34,746,520	34,043,520	34,819,365	35,610,221	36,416,390	37,236,398
11 year	**12 year**	**13 year**	**14 year**	**15 year**	**16 year**	**17 year**	**18 year**	**19 year**	**20 year**
38,074,123	30,102,557	39,798,667	40,684,295	22,967,521	42,511,374	43,451,675	44,408,368	45,385,678	46,382,051
21 year	**22 year**	**23 year**	**24 year**	**25 year**	**26 year**	**27 year**	**28 year**	**29 year**	**30 year**

47,397,873	48,431,530	49,487,430	39,631,591	51,661,584	52,778,591	53,919,601	55,082,968	56,269,144	33,141,405
31 year	**32 year**	**33 year**	**34 year**	**35 year**	**36 year**	**37 year**	**38 year**		
58,709,596	59,966,986	61,249,075	62,554,107	63,887,096	51,704,136	66,632,263	68,043,162		

Results

(1) maximizing the project value that generates the optimal capital structure;

With the criterion of maximizing the project value, the repayment amount in repayment periods are shown in Figure (1) and Table (4) with various standard deviation of revenue. The standard deviations are from 5% of total revenue up to 35% of total revenue. The results show that the higher volatility of revenue, the lower repayment amount. Due to the low repayment amount, the project could have low debt ratio and high capital structure. While σ_x is up to more than 30% of total revenue, this project cannot obtain any funds from the debt providers.

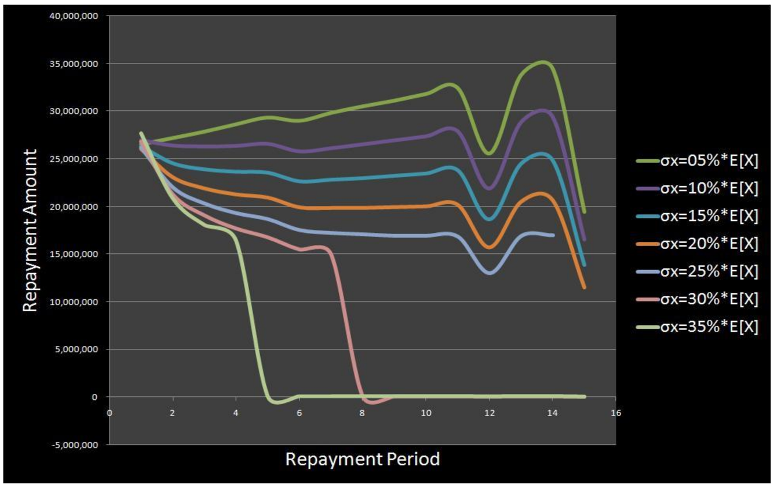

Figure 1: Debt capacity of the project with various standard deviation of revenue and with Dmax criteria.

Table 4: Debt capacity of the project with various standard deviation of revenue and with Dmax criteria. (in NT$)

σ / Repayment	5%	10%	15%	20%	25%	30%	35%
1 year	26,561,749	26,960,574	26,322,454	26,003,394	26,162,924	26,801,044	27,678,459
2 year	27,216,19	26,401,33	24,527,16	23,060,42	22,001,11	21,267,74	20,860,31

	2	6	7	6	3	3	5
3 year	27,886,836	26,305,194	23,891,110	21,893,247	20,311,606	19,062,942	18,064,010
4 year	28,657,678	26,361,662	23,640,458	21,259,405	19,303,540	17,687,825	16,412,260
5 year	29,360,810	26,581,088	23,540,768	20,934,779	18,676,255	16,765,196	86,866
6 year	29,022,101	25,787,966	22,638,941	19,915,459	17,532,413	15,489,802	85,109
7 year	29,857,606	26,114,524	22,806,684	19,847,038	17,235,586	14,972,327	87,048
8 year	30,535,765	26,529,615	22,968,593	19,852,698	17,092,906	89,026	89,026
9 year	31,136,013	26,948,129	23,215,449	19,937,973	16,933,621	91,041	91,041
10 year	31,837,120	27,368,752	23,458,931	20,014,564	16,942,561	93,091	93,091
11 year	32,458,190	27,889,295	23,796,327	20,179,285	16,847,799	95,185	95,185
12 year	25,587,174	21,899,610	18,663,585	15,728,586	13,019,356	75,256	75,256
13 year	33,828,867	28,854,034	24,476,180	20,496,314	16,914,434	99,497	99,497
14 year	34,479,940	29,394,403	24,817,420	20,647,280	16,985,693	101,711	101,711
15 year	19,464,974	16,536,615	13,895,350	11,541,179	9,416,684	57,419	57,419

(2) maximizing the debt value for obtaining the debt capacity

With the criterion of maximizing the debt value, the repayment amount in repayment periods are shown in Figure (2) and Table (5) with various standard deviation of revenue. The standard deviations are from 5% of total revenue up to 35% of total revenue. The results show that the higher volatility of revenue, the lower repayment amount. Due to the low repayment amount, the project could have low debt ratio and high capital structure. While σ_x is up to more than 30% of total revenue, this project cannot obtain any funds from the debt providers.

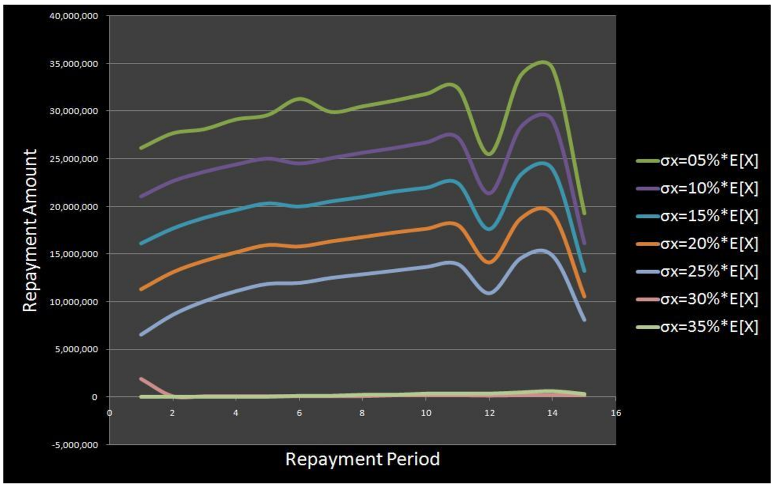

Figure 2: Debt capacity of the project with various standard deviation of revenue and with Vmax criteria.

Table 5: Debt capacity of the project with various standard deviation of revenue and with Vmax criteria. (in NT$)

σ / Repayment	5%	10%	15%	20%	25%	30%	35%
1 year	26,162,924	21,057,963	16,112,532	11,326,632	6,540,731	1,914,360	79,765
2 year	27,705,106	22,652,998	17,682,376	13,119,182	8,637,474	81,486	81,486
3 year	28,136,569	23,641,377	18,813,209	14,318,017	10,072,559	83,244	83,244
4 year	29,167,903	24,405,797	19,643,690	15,221,734	11,139,928	85,038	85,038
5 year	29,621,409	25,017,495	20,326,714	15,983,399	11,900,683	86,866	86,866
6 year	31,320,038	24,511,334	20,000,568	15,830,237	12,000,341	85,109	170,218
7 year	29,944,654	25,069,943	20,543,425	16,365,102	12,534,971	87,048	174,097
8 year	30,535,765	25,639,359	21,010,030	16,825,830	12,908,705	89,026	267,077
9 year	31,136,013	26,128,760	21,576,711	17,297,785	13,291,982	182,082	273,123
10 year	31,837,120	26,717,115	21,969,475	17,687,289	13,684,376	186,182	372,364
11 year	32,458,190	27,222,998	22,463,732	18,085,208	13,992,240	190,371	380,741
12 year	25,511,917	21,372,816	17,609,996	14,148,202	10,912,177	150,513	376,282
13 year	33,828,867	28,356,550	23,381,717	18,804,870	14,626,010	198,993	497,483
14 year	34,479,940	28,987,560	23,902,023	19,223,329	14,849,768	203,421	610,264
15 year	19,292,718	16,134,684	13,206,325	10,565,060	8,096,051	172,256	344,513

(3) value at risk (VaR) with 99% confidence level.

With the criterion of value at risk (VaR) with 99% confidence level, the repayment amount in repayment periods are shown in Figure (3) and Table (6) with various standard deviation of revenue. The standard deviations are from 5% of total revenue up to 35% of total revenue.

The results show that the higher volatility of revenue, the lower repayment amount. Due to the low repayment amount, the project could have low debt ratio and high capital structure.

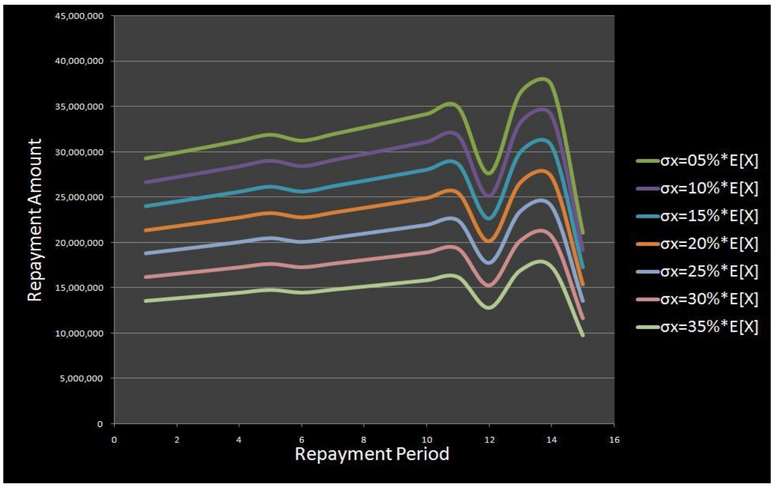

Figure 3: Debt capacity of the project with various standard deviation of revenue and with VaR criteria.

Table 6: Debt capacity of the project with various standard deviation of revenue and with VaR criteria. (in NT$)

σ / Repayment	5%	10%	15%	20%	25%	30%	35%
1 year	29,273,759	26,641,514	24,009,268	21,377,023	18,824,543	16,192,297	13,560,052
2 year	29,905,217	27,216,192	24,527,167	21,838,142	19,230,603	16,541,578	13,852,553
3 year	30,550,653	27,803,592	25,056,530	22,309,469	19,645,652	16,898,590	14,151,529
4 year	31,208,806	28,402,565	25,596,323	22,790,082	20,068,878	17,262,637	14,456,395
5 year	31,879,933	29,013,345	26,146,757	23,280,169	20,500,447	17,633,859	14,767,271
6 year	31,234,929	28,426,339	25,617,749	22,809,158	20,085,677	17,277,086	14,468,496
7 year	31,946,768	29,074,170	26,201,572	23,328,975	20,543,425	17,670,828	14,798,230
8 year	32,672,378	29,734,535	26,796,691	23,858,848	21,010,030	18,072,187	15,134,344
9 year	33,412,038	30,407,686	27,403,333	24,398,981	21,485,670	18,481,318	15,476,966
10	34,164,39	31,092,39	28,020,38	24,948,38	21,969,47	18,897,47	15,825,46

year	5	2	9	6	5	2	9
11 year	34,933,007	31,791,892	28,650,777	25,509,662	22,463,732	19,322,617	16,181,502
12 year	27,619,096	25,135,635	22,652,174	20,168,713	17,760,509	15,277,048	12,793,587
13 year	36,515,277	33,231,887	29,948,497	26,665,107	23,481,214	20,197,824	16,914,434
14 year	37,327,840	33,971,386	30,614,932	27,258,477	24,003,734	20,647,280	17,290,825
15 year	21,072,701	19,177,880	17,283,060	15,388,239	13,550,837	11,656,017	9,761,196

CONCLUSIONS

Three criteria were adopted in the study: (1) maximizing the project value that generates the optimal capital structure; (2) maximizing the debt value for obtaining the debt capacity; and (3) value at risk with 99% confidence level. The debt capacity and the capital structure of the project are similar by using the criteria of (1) maximizing the project value that generates the optimal capital structure; (2) maximizing the debt value for obtaining the debt capacity. The third criterion of VaR leads to a little higher of debt capacity and the capital structure of the project

The standard deviations are from 5% of total revenue up to 35% of total revenue. The results show that the higher volatility of revenue, the lower repayment amount. Due to the low repayment amount, the project could have low debt ratio and high capital structure. While σ_x is up to more than 30% of total revenue, this project cannot obtain any funds from the debt providers.

REFERENCE

- Dias, Antonio Jr., Ioannou, Photios G., Debt Capacity and Optimal Capital Structure for Privately Financed Infrastructure Project, Journal of Construction Engineering and Management, Vol. 121, No. 4, December 1995, 404-414.
- .Sandalkhan Bakatjan, Metin Arikan, and Robert L.K. Tiong, 2003, “Optimal capital structure model for BOT power projects in Turkey”, Journal of Construction Engineering and Management, Vol. 129, No.1, pp. 89~97.(2003)

- Wibowo, A., 2006, CAPM-Based Valuation of Financial Government Supports to infeasible and Risky Private Infrastructure Projects, J. of Construction Engineering and Management, Vol. 132, No. 3, pp. 239-248.
- Zayed, T. M., and Chang, L. M., 2002,Prototype Model for Build-Operate-Transfer Risk Assessment, J. of Management in Engineering, Vol. 18, No. 1, pp. 7-16.
- Zhang, X. 2005,Financial Viability Analysis and Capital Structure Optimization in Privatized Public Infrastructure Projects, Journal of Construction Engineering and Management, Vol. 131, No.6, pp. 656-668.
- Zhang, X., 2006,Factor Analysis of Public Clients' Best-Value Objective in Public-Privately Partnered Infrastructure Projects, J. of Construction Engineering and Management, Vol. 132, No. 9, pp. 956-965.